Given $P(X_t | e_{1:t})$
also $P(y_0) P(X_{t+1} | X_t) P(e_t | X_t)$

How to get $P(X_{t+1} | e_{1:t})$?

\[
P(x_{t+1} | e_{1:t}) = \sum_{x_t} P(x_{t+1}, x_t | e_{1:t}) = \sum_{x_t} \frac{P(x_{t+1}, x_t | e_{1:t})}{P(e_t | x_t)} = \sum_{x_t} \frac{P(x_{t+1} | x_t, e_{1:t}) P(x_t | e_{1:t})}{P(x_t | e_{1:t})} = \sum_{x_t} P(x_{t+1} | x_t, e_{1:t}) = \sum_{x_{t+1}} P(x_{t+1} | e_{1:t})
\]

Independence

$X_t \perp X_1, \ldots, X_{t-2} | X_{t-1}$

$X_t \perp E_1, \ldots, E_{t-1} | X_{t-1}$

$E_t \perp X_1, \ldots, X_{t-1} | X_t$

$E_t \perp E_1, \ldots, E_{t-1} | X_t$

$P(X_t | x_{0:t-1}) = P(X_t | x_{t-1})$

$P(E_t | x_{0:t}; E_{0:t-1}) = P(E_t | X_t)$
Weather HMM

$B_0(\tau, -r) = P(R_0) = <0.5, 0.5>$ initialization

\[\text{time pass}\]
$B_0'(\tau, -r) = P(R_1) = \sum_{R_0} P(R_1|R_0)P(R_0)\]
\[= <0.7, 0.3> \times 0.5 + <0.3, 0.7> \times 0.5\]
\[= <0.5, 0.5>\]

\[\text{observe an evidence } U_1 = \text{true (umbrella appears)}\]

$B_1(\tau, -r) = P(R_1|U_1) = \frac{P(R_1, U_1)}{P(U_1)} = \sum_{R_1} P(R_1, U_1) P(R_1) = P(U_1|R_1)P(R_1) = <0.9, 0.2> <0.5, 0.5> = <0.45, 0.1> \approx <0.818, 0.182> \text{ so, sum to 1}\]

\[\text{time pass}\]
$B_1'(\tau, -r) = P(R_2|U_1) = \sum_{R_1} P(R_2|R_1)P(R_1|U_1) = <0.7, 0.3> \times 0.818 + <0.3, 0.7> \times 0.182 \approx <0.627, 0.373>\]

\[\text{observe another evidence } U_2 = \text{true (umbrella appears again)}\]

$B_2(\tau, -r) = P(R_2|U_1, U_2) = \ldots \ldots$