the clients and the associates are enemies.

los clientes y los asociados son enemigos.

the company has three groups.

la empresa tiene tres grupos.

its groups are in Europe.

sus grupos estan en Europa.

the modern groups sell strong pharmaceuticals.

los grupos modernos venden medicinas fuertes.

the groups do not sell zanzanine.

los grupos no venden zanzanina.

the small groups are not modern.

los grupos pequenos no son modernos.
García y asociados.

Carlos García tiene tres asociados.

Sus asociados no son fuertes.

García también tiene una empresa.

Sus clientes están enfadados.

Los asociados también están enfadados.

Carlos García has three associates.

The clients and the associates are enemies.

The company has three groups.

La empresa tiene tres grupos.

Its groups are in Europe.

Sus grupos están en Europa.

The modern groups sell strong pharmaceuticals.

Los grupos modernos venden medicinas fuertes.

The groups do not sell zanazaine.

Los grupos no venden zanazaina.

The small groups are not modern.

Los grupos pequeños no son modernos.
Australia is one of the few countries that have diplomatic relations with North Korea.
he
it
, it
, he
it is
he will be
it goes
he goes
is
are
go
es
is
yes
, of course
not
do not
does not
is not
after
to
according to
in
house
home
chamber
at home
under house
return home
do not
is
are
is after all
does
not
is not
are not
is not a
follow
not after
not to
er geht ja nicht nach hause

he goes yes not to home
CS 5522: Artificial Intelligence II

Adversarial Search

Instructor: Wei Xu
Ohio State University

[These slides were adapted from CS188 Intro to AI at UC Berkeley.]
Game Playing State-of-the-Art
Game Playing State-of-the-Art

- **Checkers**: 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!
Game Playing State-of-the-Art

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!
Game Playing State-of-the-Art

- **Checkers**: 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!

- **Chess**: 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.
Game Playing State-of-the-Art

- **Checkers**: 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!

- **Chess**: 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.
**Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!

**Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

**Go:** Human champions are now starting to be challenged by machines, though the best humans still beat the best machines. In go, b > 300! Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.
**Game Playing State-of-the-Art**

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!

- **Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

- **Go:** Human champions are now starting to be challenged by machines, though the best humans still beat the best machines. In go, $b > 300$! Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.

Chess: 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

Go: Human champions are now starting to be challenged by machines, though the best humans still beat the best machines. In go, b > 300! Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.
Game Playing State-of-the-Art

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!

- **Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

- **Go:** Human champions are now starting to be challenged by machines, though the best humans still beat the best machines. In go, b > 300! Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.

- **Pacman**
**Game Playing State-of-the-Art**

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!

- **Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

- **Go:** Human champions are now starting to be challenged by machines, though the best humans still beat the best machines. In go, $b > 300$! Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.

- **Pacman**
Behavior from Computation

[Demo: mystery pacman (L6D1)]
Adversarial Games
Many different kinds of games!

Axes:
- Deterministic or stochastic?
- One, two, or more players?
- Zero sum?
- Perfect information (can you see the state)?

Want algorithms for calculating a strategy (policy) which recommends a move from each state
Deterministic Games

- Many possible formalizations, one is:
  - States: $S$ (start at $s_0$)
  - Players: $P=\{1\ldots N\}$ (usually take turns)
  - Actions: $A$ (may depend on player / state)
  - Transition Function: $S \times A \rightarrow S$
  - Terminal Test: $S \rightarrow \{t,f\}$
  - Terminal Utilities: $S \times P \rightarrow \mathbb{R}$

- Solution for a player is a policy: $S \rightarrow A$
Zero-Sum Games

- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition

- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible
  - More later on non-zero-sum games
Adversarial Search
Single-Agent Trees
Single-Agent Trees
Single-Agent Trees
Single-Agent Trees
Single-Agent Trees

The diagram illustrates a single-agent decision tree with multiple possible outcomes. Each decision node branches into different paths, each represented by a different agent or option. The tree structure shows the progression of decisions and potential outcomes, with numbers indicating possible values at each branching point.
Value of a State

2 0 ... 2 6 ... 4 6

8
Value of a state: The best achievable outcome (utility) from that state.
Value of a State

**Value of a state:**
The best achievable outcome (utility) from that state

**Terminal States:**
\[ V(s) = \text{known} \]
Value of a State

Value of a state: The best achievable outcome (utility) from that state

Non-Terminal States:

\[ V(s) = \max_{s' \in \text{children}(s)} V(s') \]

Terminal States:

\[ V(s) = \text{known} \]
Adversarial Game Trees
Adversarial Game Trees
Adversarial Game Trees
Adversarial Game Trees
Adversarial Game Trees
Minimax Values

-8  -5  -10  +8
Minimax Values

Terminal States:
\[ V(s) = \text{known} \]
Minimax Values

States Under Opponent’s Control:

\[ V(s') = \min_{s \in \text{successors}(s')} V(s) \]

Terminal States:

\[ V(s) = \text{known} \]
Minimax Values

**States Under Agent’s Control:**

\[ V(s) = \max_{s' \in \text{successors}(s)} V(s') \]

**States Under Opponent’s Control:**

\[ V(s') = \min_{s \in \text{successors}(s')} V(s) \]

**Terminal States:**

\[ V(s) = \text{known} \]
Tic-Tac-Toe Game Tree

MAX (X)

MIN (O)

MAX (X)

MIN (O)

TERMINAL

Utility

-1 0 +1
Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node’s minimax value: the best achievable utility against a rational (optimal) adversary
Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node’s minimax value: the best achievable utility against a rational (optimal) adversary

```
8  2  5  6

max
min

Terminal values: part of the game
```
Adversarial Search (Minimax)

- **Deterministic, zero-sum games:**
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

- **Minimax search:**
  - A state-space search tree
  - Players alternate turns
  - Compute each node’s minimax value: the best achievable utility against a rational (optimal) adversary
Adversarial Search (Minimax)

- **Deterministic, zero-sum games:**
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

- **Minimax search:**
  - A state-space search tree
  - Players alternate turns
  - Compute each node’s **minimax value:** the best achievable utility against a rational (optimal) adversary

```
                      max
                     /   \
                  /     \
                2       5
               / \     /  
              8   2   5   6
```

Minimax values: computed recursively

Terminal values: part of the game
Adversarial Search (Minimax)

- **Deterministic, zero-sum games:**
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

- **Minimax search:**
  - A state-space search tree
  - Players alternate turns
  - Compute each node’s minimax value: the best achievable utility against a rational (optimal) adversary

Minimax values: computed recursively

Terminal values: part of the game
Minimax Implementation

def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
Minimax Implementation

def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v

def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
def max-value(state):
    initialize v = -\infty
    for each successor of state:
        v = max(v, min-value(successor))
    return v

def min-value(state):
    initialize v = +\infty
    for each successor of state:
        v = min(v, max-value(successor))
    return v
Minimax Implementation

**Defining the Min-Value Function**

```python
def min_value(state):
    initialize v = +\infty
    for each successor of state:
        v = min(v, max_value(successor))
    return v
```

**Defining the Max-Value Function**

```python
def max_value(state):
    initialize v = -\infty
    for each successor of state:
        v = max(v, min_value(successor))
    return v
```

**Bellman's Equations**

- $V(s) = \max_{s' \in \text{successors}(s)} V(s')$
- $V(s') = \min_{s \in \text{successors}(s')} V(s)$
def value(state):
    if the state is a terminal state: return the state’s utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
Minimax Implementation (Dispatch)

```python
def value(state):
    if the state is a terminal state: return the state’s utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)

def max-value(state):
    initialize $v = -\infty$
    for each successor of state:
        $v = \max(v, \text{value(successor)})$
    return $v$

def min-value(state):
    initialize $v = +\infty$
    for each successor of state:
        $v = \min(v, \text{value(successor)})$
    return $v$
```
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example

3  12  8
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example

Diagram showing a minimax tree with values at each node and arrows indicating the decision path.
Minimax Example

```
     12
   /   \
  8     5
 /   \ /   \
3     2 4     6
     /   \     /   \
    14   5   2   
```

Minimax Efficiency

- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?
Minimax Properties

Optimal against a perfect player. Otherwise?

[Demo: min vs exp (L6D2, L6D3)]
Minimax Properties

Optimal against a perfect player. Otherwise?

[Demo: min vs exp (L6D2, L6D3)]
Minimax Properties

Optimal against a perfect player. Otherwise?

[Demo: min vs exp (L6D2, L6D3)]
Video of Demo Min vs. Exp (Min)
Video of Demo Min vs. Exp (Min)
Video of Demo Min vs. Exp (Min)
Video of Demo Min vs. Exp (Exp)
Video of Demo Min vs. Exp (Exp)
Video of Demo Min vs. Exp (Exp)
Resource Limits
Resource Limits

- Problem: In realistic games, cannot search to leaves!
Resource Limits

- **Problem**: In realistic games, cannot search to leaves!
- **Solution**: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions
Resource Limits

- **Problem:** In realistic games, cannot search to leaves!
- **Solution:** Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions

![Diagram of a search tree with min and max values.](image)
Resource Limits

- **Problem:** In realistic games, cannot search to leaves!
- **Solution:** Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions
Problem: In realistic games, cannot search to leaves!

Solution: Depth-limited search
- Instead, search only to a limited depth in the tree
- Replace terminal utilities with an evaluation function for non-terminal positions

Example:
- Suppose we have 100 seconds, can explore 10K nodes / sec
- So can check 1M nodes per move
- $\alpha$-$\beta$ reaches about depth 8 - decent chess program
Resource Limits

- **Problem:** In realistic games, cannot search to leaves!

- **Solution:** Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions

- **Example:**
  - Suppose we have 100 seconds, can explore 10K nodes/sec
  - So can check 1M nodes per move
  - $\alpha$-$\beta$ reaches about depth 8 - decent chess program

- **Guarantee of optimal play is gone**

- **More plies makes a BIG difference**

- **Use iterative deepening** for an anytime algorithm
Depth Matters

- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation

[Demo: depth limited (L6D4, L6D5)]
Video of Demo Limited Depth (2)
Video of Demo Limited Depth (2)
Video of Demo Limited Depth (2)
Video of Demo Limited Depth (10)
Video of Demo Limited Depth (10)
Video of Demo Limited Depth (10)
Evaluation Functions
Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search

- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

\[ \text{Eval}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s) \]

- e.g. \( f_1(s) = (\text{num white queens} - \text{num black queens}) \), etc.
Evaluation for Pacman

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate]
Evaluation for Pacman

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate]
Evaluation for Pacman

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate (L6D6,7,8,10)]
Evaluation for Pacman

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate]
Evaluation for Pacman

[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate]
Video of Demo Thrashing (d=2)
Video of Demo Thrashing (d=2)
Video of Demo Thrashing (d=2)
Why Pacman Starves

- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!
A danger of replanning agents!
- He knows his score will go up by eating the dot now (west, east)
- He knows his score will go up just as much by eating the dot later (east, west)
- There are no point-scoring opportunities after eating the dot (within the horizon, two here)
- Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!
Why Pacman Starves

- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!
A danger of replanning agents!
- He knows his score will go up by eating the dot now (west, east)
- He knows his score will go up just as much by eating the dot later (east, west)
- There are no point-scoring opportunities after eating the dot (within the horizon, two here)
- Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!
Video of Demo Smart Ghosts (Coordination)
Video of Demo Smart Ghosts (Coordination)
Video of Demo Smart Ghosts (Coordination)
Video of Demo Smart Ghosts (Coordination) - Zoomed In
Video of Demo Smart Ghosts (Coordination) - Zoomed In
Video of Demo Smart Ghosts (Coordination) - Zoomed In
Game Tree Pruning
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Example
Minimax Pruning
Minimax Pruning
Minimax Pruning
Minimax Pruning
Minimax Pruning
Minimax Pruning
Minimax Pruning
Minimax Pruning
Minimax Pruning
Minimax Pruning
Minimax Pruning
Alpha-Beta Pruning

- **General configuration (MIN version)**
  - We’re computing the MIN-VALUE at some node $n$
  - We’re looping over $n$’s children
  - $n$’s estimate of the children’s’ min is dropping
  - Who cares about $n$’s value? MAX
  - Let $a$ be the best value that MAX can get at any choice point along the current path from the root
  - If $n$ becomes worse than $a$, MAX will avoid it, so we can stop considering $n$’s other children (it’s already bad enough that it won’t be played)

- **MAX version is symmetric**
def min-value(state, α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v

def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v

α: MAX’s best option on path to root
β: MIN’s best option on path to root
Minimax Implementation (Recap)

```python
def value(state):
    if the state is a terminal state: return the state’s utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```
**Minimax Implementation (Recap)**

```python
def value(state):
    if the state is a terminal state: return the state’s utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)

def max-value(state):
    initialize v = -\infty
    for each successor of state:
        v = max(v, value(successor))
    return v

def min-value(state):
    initialize v = +\infty
    for each successor of state:
        v = min(v, value(successor))
    return v
```
- This pruning has no effect on minimax value computed for the root!

- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won’t let you do action selection
Alpha-Beta Pruning Properties

- This pruning has **no effect** on minimax value computed for the root!

- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won’t let you do action selection

- Good child ordering improves effectiveness of pruning

- With “perfect ordering”:
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless...

- This is a simple example of **metareasoning** (computing about what to compute)
Alpha-Beta Quiz

Diagram:
- Node a
- Node h
- Node b
- Node e
- Node i
- Node l
- Node c
- Node d
- Node f
- Node g
- Node j
- Node k
- Node m
- Node n
- Leaves: 10, 6, 100, 8, 1, 2, 20, 4