1 Joint and Conditional Distributions

Consider three random variables Toothache, Cavity and Catch. The joint probabilities that each random variable takes on the respective values is given below:

<table>
<thead>
<tr>
<th></th>
<th>+toothache</th>
<th>−toothache</th>
</tr>
</thead>
<tbody>
<tr>
<td>+catch</td>
<td>0.108</td>
<td>0.012</td>
</tr>
<tr>
<td>−catch</td>
<td>0.016</td>
<td>0.064</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>+cavity</th>
<th>−cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>+catch</td>
<td>0.072</td>
<td>0.008</td>
</tr>
<tr>
<td>−catch</td>
<td>0.144</td>
<td>0.576</td>
</tr>
</tbody>
</table>

Questions:
1) What is $P(+\text{toothache})$?

2) What is $P(\text{Catch})$?

3) What is $P(+\text{cavity} | +\text{catch})$?

4) What is $P(+\text{cavity} | +\text{toothache or } +\text{catch})$?

5) Is the random variable Catch conditionally independent of Toothache, given Cavity? (Hint: $P(+\text{catch} | +\text{toothache}, +\text{cavity}) = P(+\text{catch} | +\text{cavity})$ ?)
2 Conditional Independence

For random variables $X, Y, Z$, show that the following three statements are equivalent:

(i) $P(X|Y, Z) = P(X|Z)$
(ii) $P(Y|X, Z) = P(Y|Z)$
(iii) $P(X, Y|Z) = P(X|Z)P(Y|Z)$

Equivalence of the first two statements show that conditional independence is symmetric ($X$ and $Y$ are conditionally independent given $Z$, and the order of $X$ and $Y$ doesn’t matter). The third statement is analogous to the definition of unconditional independence: $P(X, Y) = P(X)P(Y)$.