1 Blackjack

In this question, you will play a simplified version of blackjack where the deck is infinite and the dealer always has a fixed count of 15. The deck contains cards 2 through 10, J, Q, K, and A, each of which is equally likely to appear when a card is drawn. Each number card is worth the number of points shown on it, the cards J, Q, and K are worth 10 points, and A is worth 11. At each turn, you may either hit or stay. If you choose to hit, you receive no immediate reward and are dealt an additional card. If you stay, you receive a reward of 0 if your current point total is exactly 15, +10 if it is higher than 15 but not higher than 21, and −10 otherwise (i.e. lower than 15 or larger than 21). After taking the stay action, the game enters a terminal state end and ends. A total of 22 or higher is referred to as a bust; from a bust, you can only choose the action stay. As your state space you take the set \{0, 2, \ldots, 21, \text{bust}, \text{end}\} indicating point totals, “bust” if your point total exceeds 21, and “end” for the end of the game.

Questions:
1) Suppose you have performed \(k\) iterations of value iteration. Compute \(V_{k+1}(12)\) given the partial table below for \(V_k(s)\). Give your answer in terms of the discount \(\gamma\) as a variable. Note: do not worry about whether the listed \(V_k\) values could actually result from this MDP!
\[ V_{k+1}(s) = \max_a \sum_{s'} T(s, a, s') (R(s, a, s') + \gamma V_k(s')) \]

2. You suspect that the cards do not actually appear with equal probability and decide to use Q-learning instead of value iteration. Given the partial table of initial Q-values below, fill in the partial table of Q-values on the right after the following episode occurred. Assume a learning rate of 0.5 and a discount factor of 1. The initial portion of the episode has been omitted. Leave blank any values which Q-learning does not update.

### Initial values

<table>
<thead>
<tr>
<th>( s )</th>
<th>( a )</th>
<th>( Q(s, a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>hit</td>
<td>-2</td>
</tr>
<tr>
<td>19</td>
<td>stay</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>hit</td>
<td>-4</td>
</tr>
<tr>
<td>20</td>
<td>stay</td>
<td>7</td>
</tr>
<tr>
<td>21</td>
<td>hit</td>
<td>-6</td>
</tr>
<tr>
<td>21</td>
<td>stay</td>
<td>8</td>
</tr>
<tr>
<td>bust</td>
<td>stay</td>
<td>-8</td>
</tr>
</tbody>
</table>

### Episode

<table>
<thead>
<tr>
<th>( s )</th>
<th>( a )</th>
<th>( r )</th>
<th>( s' )</th>
<th>( a' )</th>
<th>( r' )</th>
<th>( s'' )</th>
<th>( a'' )</th>
<th>( r'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>hit</td>
<td>0</td>
<td>21</td>
<td>hit</td>
<td>0</td>
<td>bust</td>
<td>stay</td>
<td>-10</td>
</tr>
</tbody>
</table>

### Updated values

<table>
<thead>
<tr>
<th>( s )</th>
<th>( a )</th>
<th>( Q(s, a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>hit</td>
<td>3</td>
</tr>
<tr>
<td>19</td>
<td>stay</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>hit</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>stay</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>hit</td>
<td>-7</td>
</tr>
<tr>
<td>21</td>
<td>stay</td>
<td></td>
</tr>
<tr>
<td>bust</td>
<td>stay</td>
<td>-9</td>
</tr>
</tbody>
</table>

\[ Q(19, \text{hit}) \leftarrow (1-\alpha)Q(19, \text{hit}) + \alpha \cdot (R(s, a, r) + \gamma \max_{a'} Q(s', a')) \]

\[ Q(19, \text{hit}) = (1-0.5) \times -2 + 0.5 \times (0 + 1 \times \max(-6, 8)) = 3 \]