1. **Language Identification.** The Naive Bayes model has been famously used for text classification. In this case, we will use it in the bag-of-words model to determine the language of Twitter posts:

- Each tweet has binary class label $C$ which takes values in $\{sp, en\}$. The $sp$ stands for Spanish, $en$ stands for English.
- For a tweet with $n$ words $t_1, \ldots, t_n$, its label is predicted by
  \[
  \arg\max_c P(C = c | t_1, \ldots, t_n) = \arg\max_c P(C = c) \prod_{i=1}^n P(W = t_i | C = c)
  \]
- Each word $t$ of a tweet, no matter where in the tweet the word occurs, is assumed to have probability $P(W = t | C)$.

You are given four tweets as a training set, and one new tweet to classify:

<table>
<thead>
<tr>
<th>Training</th>
<th>#1</th>
<th>English Wikipedia editor</th>
<th>en</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#2</td>
<td>free English Wikipedia</td>
<td>$en$</td>
</tr>
<tr>
<td></td>
<td>#3</td>
<td>Wikipedia editor</td>
<td>$en$</td>
</tr>
<tr>
<td></td>
<td>#4</td>
<td>españo de Wikipedia</td>
<td>$sp$</td>
</tr>
<tr>
<td>Test</td>
<td>#5</td>
<td>Wikipedia españo el</td>
<td>??</td>
</tr>
</tbody>
</table>

(a) What values would you estimate for the maximum likelihood parameters for the Naive Bayes model, if not using any smoothing? (Note: Only the parameters that would be involved in the prediction for tweet #5 are listed here.)

\[
\hat{P}(C)
\]

\[
\hat{P}(W = t | C = en)
\]

\[
\hat{P}(W = t | C = sp)
\]
What is the probability of tweet #5 being predicted as English or Spanish by this Naive Bayes model?

\[ P(en|\text{Wikipedia, español, el}) = \]

\[ P(sp|\text{Wikipedia, español, el}) = \]

(b) You are training with the same tweets, but now doing Laplace Smoothing with strength \( k = 1 \). Re-estimate the parameters. How will this new Naive Bayes model will classify tweet #5?

\[
\hat{P}(C) \quad \hat{P}(W = t|C = en) \quad \hat{P}(W = t|C = sp)
\]

<table>
<thead>
<tr>
<th></th>
<th>Wikipedia</th>
<th>español</th>
<th>el</th>
</tr>
</thead>
<tbody>
<tr>
<td>en</td>
<td></td>
<td></td>
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<tr>
<td>sp</td>
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</tbody>
</table>

\[ P(en|\text{Wikipedia, español, el}) = \]

\[ P(sp|\text{Wikipedia, español, el}) = \]