CSE 5522 Artificial Intelligence II

Homework #6: Hidden Markov Models

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At time t, Peter is in some state X_t. The two states Peter alternates between are saving the world (denoted as S) and being a CSE student (denoted as C). Let the evidence E_t be whether or not Peter is seen in the CSE labs at time t.

The transition probabilities are provided in the following table (left), where the row corresponds to X_{t-1} and the column to X_t. For example, $P(X_t = S|X_{t-1} = C) = 0.4$.

X_{t-1}	X_t	$P(X_t\|X_{t-1})$
C	C	0.6
C	S	0.4
S	C	0.2
S	S	0.8

X_t	E_t	$P(E_t\|X_t)$
C	true	0.7
C	false	0.3
S	true	0.1
S	false	0.9

The model for evidence E_t is provided in the following table, where the row corresponds to X_t and the column to E_t. For example, $P(E_t = false|X_t = S) = 0.9$.

Questions:

1) Assume we have current belief $B(X_t) = P(X_t|e_{i:t})$, how to compute for passage of time?

$$B'(X_{t+1}) = P(X_{t+1}|e_{1:t}) =$$

How to update the belief with the observation of evidence (forward algorithm)?

$$B(X_{t+1}) = P(X_{t+1}|e_{1:t+1}) \propto$$

2) Let the initial beliefs be $P(X_0 = S) = P(X_0 = C) = 0.5$. Fill in the following table for $t = 1$, first computing for passage of time, and then for observation of evidence $E_1 = true$. Finally, normalize the values from the observation column to get the beliefs. Round to three decimal places.

X_1	passage of time $B'(X_1)$	update with evidence $E_1 = true$	$B(X_1)$
S			
C			

3) Repeat for $t = 2$, with the observation of evidence $E_2 = false$. When using any previous value for computations, use their rounded value. Round to three decimal places.

X_2	passage of time $B'(X_2)$	update with evidence $E_2 = false$	$B(X_2)$
S			
C			

4) Assume now we are using a particle filter with 3 particles to apprximate our belief instead of using exact inference as in 1 – 3). Image we have just applied transition model sampling (passage of time) from state X_0 to X_1, and now have the set of particles S, S, C. What is our belief about X_1 before considering noisy evidence?

X_1	passage of time $B'(X_1)$
S	
C	

5) Now assume we receive evidence $E_1 = true$. What is the weight for each particle, and what is our belief now about X_1 (before weighted re-sampling)?

particle	weight
S	
S	
C	

X_1	after observation $B(X_1)$
S	
C	

The last two questions are optional (up to 1% bonus).

6) Will performing weighted re-sampling on these weighted particles to obtain our three new particle representation for X_1 cause our belief to change?

7) Recall from the lecture that $m_t[x_t] = max_{x_{1:t-1}} P(x_{1:t-1}, x_t, e_{1:t})$, which is the probability of the most likely path that ends at x_t, considering the path up to t and the evidence up to t. Use **Viterbi algorithm** to compute $m_1[S_1]$, $m_1[C_1]$, $m_2[S_2]$, $m_2[C_2]$ for the sequence of evidence $E_1 = true, E_2 = false$. Define $m_0[S] = m_0[C] = 0.5$. Use exact numbers in your calculations and answers. What was Peter most likely doing at time $t = 1$ and at time $t = 2$?