CSE 5525 Artificial Intelligence II Homework #5: Probability Wei Xu, Ohio State University

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1 Joint and Conditional Distributions

Consider three random variables Toothache, Cavity and Catch. The joint probabilities that each random variable takes on the respective values is given below:

	+toothache		-toothache	
	+ catch	-catch	+ catch	-catch
+cavity	0.108	0.012	0.072	0.008
-cavity	0.016	0.064	0.144	0.576

Questions:

1) What is P(+toothache)?

2) What is P(Catch)?

3) What is P(+cavity|+catch)?

4) What is P(+cavity| + toothache or + catch)?

5) Is the random variable Catch *conditionally independent* of Toothache, given Cavity? (Hint: P(+catch| + toothache, +cavity) = P(+catch| + cavity)?)

2 Conditional Independence

For random variables X, Y, Z, show that the following three statements are equivalent:

 $\begin{array}{l} (\mathrm{i}) \ P(X|Y,Z) = P(X|Z) \\ (\mathrm{ii}) \ P(Y|X,Z) = P(Y|Z) \\ (\mathrm{iii}) \ P(X,Y|Z) = P(X|Z)P(Y|Z) \end{array}$

Equivalence of the first two statements show that conditional independence is symmetric (X and Y are conditionally independent given Z, and the order of X and Y doesn't matter). The third statement is analogous to the definition of *unconditional* independence: P(X,Y) = P(X)P(Y).