CS 7650: Natural Language Processing
Fall 2022
Problem Set 2

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Piazza: https://piazza.com/class/l6vgipz0vsm1kk
Gradescope: https://gradescope.com/courses/418978

Due: Friday, October 7, 11:59 PM ET

1 Understanding Word2Vec

Given a sequence of words $w_1, \ldots, w_T$ and context size c, the training objective of skip-gram is:

$$\mathcal{L} = -\frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log P\left(w_{t+j} \mid w_t\right)$$

where $P\left(w_o \mid w_t\right)$ is defined as:

$$P\left(w_o \mid w_t\right) = \frac{\exp\left(\mathbf{u}_{w_t}^{\top} \mathbf{v}_{w_o}\right)}{\sum_{k \in V} \exp\left(\mathbf{u}_{w_t}^{\top} \mathbf{v}_k\right)}$$

where $\mathbf{u}_k$ represents the "target" vector and $\mathbf{v}_k$ represents the"context" vector, for every $k \in V$.

(a) (**3 pts**) Derive the following gradient (probability w.r.t context vector):

$$-\frac{\partial \log P\left(w_o \mid w_t\right)}{\partial \mathbf{v}_{w_o}}$$

(b) (**2 pts**) Imagine that we train the model on a large corpus (e.g. English Wikipedia). Describe the effects of context size c to the resulting word vectors $\mathbf{u}_w$, i.e. what if we use context size $c = 1, 5$, or 100?

2 Hidden Markov Models and the Viterbi Algorithm

We have a toy language with 2 words - "cool" and "shade". We want to tag the parts of speech in a test corpus in this toy language. There are only 2 parts of speech — NN (noun) and VB (verb) in this language. We have a corpus of text in which we the following distribution of the 2 words:

	NN	VB
cool	3	6
shade	7	4

Assume that we have an HMM model with the following transition probabilities (* is a special start of the sentence symbol).

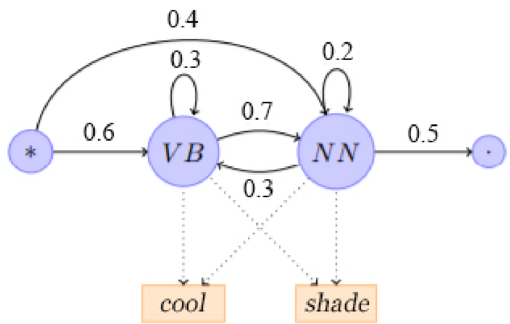


Figure 1: HMM model for POS tagging in our toy language.

(a) (**2 pts**) Compute the emission probabilities for each word given each POS tag.

(b) (**3 pts**) Draw the Viterbi trellis for the sequence "cool shade.". Highlight the most likely sequence. Here is an example of Viterbi trellis.

3 LSTMs

The update equations for a LSTM at timestep i are given in the following equations. Eisenstein Chapter 6.3 may be useful in answering this question.

$$
\begin{aligned}
\mathbf{f_i} &= \sigma(\mathbf{W}^{(f)}\mathbf{h_{i-1}} + \mathbf{U}^{(f)}\mathbf{x_i} + \mathbf{B}^{(f)}) && \text{Forget gate} \\
\mathbf{i_i} &= \sigma(\mathbf{W}^{(i)}\mathbf{h_{i-1}} + \mathbf{U}^{(i)}\mathbf{x_i} + \mathbf{B}^{(i)}) && \text{Input gate} \\
\mathbf{g_i} &= tanh(\mathbf{W}^{(g)}\mathbf{h_{i-1}} + \mathbf{U}^{(g)}\mathbf{x_i}) && \text{Update candidate} \\
\mathbf{c_i} &= \mathbf{f_i} \odot \mathbf{c}_{i-1} + \mathbf{i_i} \odot \mathbf{g_i} && \text{Memory cell update} \\
\mathbf{o_i} &= \sigma(\mathbf{W}^{(o)}\mathbf{h_{i-1}} + \mathbf{U}^{(o)}\mathbf{x_i} + \mathbf{B}^{(o)}) && \text{Output gate} \\
\mathbf{h_i} &= \mathbf{o_i} \cdot \tanh(\mathbf{c_i}) && \text{Output}
\end{aligned}
$$

(a) (**1 pt**) In Table 1 we provide weight values and in Table 2 timestep inputs. We'll now compute the value of $\boldsymbol{h}_i$ using Table 1 and Table 2:

$\boldsymbol{f}_i = \sigma(4 + 4 + 0) = 1.0$
$\boldsymbol{i}_i = \sigma(-1 + 9 + 1) = 1.0$
$\boldsymbol{g}_i = \tanh([4, -8, -4]^T + [-3, 12, 1]^T) = \tanh([1, 4, -3]^T) = [0.76, 1.0, -1.0]^T$
$\boldsymbol{c}_i = 1.0 \odot [1, 0, -4]^T + 1.0 \odot [0.76, 1.0, -1.0]^T = [1.76, 1.0, -5.0]^T$
$\boldsymbol{o}_i = \sigma(2 + 2 - 1) = 1.0$
$\boldsymbol{h}_i = 1.0 \odot \tanh([1.76, 1.0, -5.0]^T) = \mathbf{[0.94, 0.76, -1.0]^T}$

The gates of this LSTM do not restrict the flow of any information. To effectively turn this LSTM into an Elman RNN at the current timestep, i.e., include **only** information from the current input and prior hidden state and **no** information from the prior memory cell in $\boldsymbol{h}_i$, describe the values that you would need to set the gates $\boldsymbol{f}_i, \boldsymbol{i}_i$ and $\boldsymbol{o}_i$ equal to. Only the values for these gates are necessary, do not change the equations for the update.

(b) (**1 pt**) Which variable from the list of intermediate variables in the given equations most closely resembles the hidden state of a standard Elman RNN? (Answer choices are $\boldsymbol{f}_i$, $\boldsymbol{i}_i$, $\boldsymbol{g}_i$, $\boldsymbol{c}_i$, $\boldsymbol{o}_i$, $\boldsymbol{h}_i$).

Weight	Value
$\mathbf{W}^{(f)}$	$[1, -2, -3]$
$\mathbf{U}^{(f)}$	$[0, -1, -2]$
$\mathbf{B}^{(f)}$	0
$\mathbf{W}^{(i)}$	$[0, 0, 1]$
$\mathbf{U}^{(i)}$	$[-1, -2, -2]$
$\mathbf{B}^{(i)}$	1
$\mathbf{W}^{(g)}$	$\begin{bmatrix} 0 & 1 & -3 \\ -3 & 1 & 0 \\ -2 & -1 & -3 \end{bmatrix}$
$\mathbf{U}^{(g)}$	$\begin{bmatrix} 1 & 0 & 0 \\ -2 & -3 & 0 \\ 1 & -1 & -2 \end{bmatrix}$
$\mathbf{W}^{(o)}$	$[1, 0, 1]$
$\mathbf{U}^{(o)}$	$[-1, 0, 1]$
$\mathbf{B}^{(o)}$	-1

Table 1: Weights for LSTM.

Vector	Value
$\boldsymbol{h}_{i-1}$	$[3, 1, -1]^T$
$\boldsymbol{c}_{i-1}$	$[1, 0, -4]^T$
$\boldsymbol{x}_i$	$[-3, -2, -1]^T$

Table 2: Input/intermediate variables for LSTM.

(c) (**2 pts**) In this problem, all the LSTM gates are scalars. What changes would have to be made to Table 1 in order to create vector gates? (Specify which weights would change and what their new dimensions would be). What is the benefit of vector gates over scalars?

(d) (**2 pts**) What two problems in RNNs does the inclusion of the memory cell $\boldsymbol{c}_i$ improve? What property of its computation allows it to do this?

4 Conditional Random Fields

Consider a sequential CRF,

$$\mathrm{P}(\mathbf{y}|\mathbf{x}) = \frac{1}{Z} \prod_{i=2}^{N} \exp(\phi_t(y_{i-1}, y_i)) \prod_{i=1}^{N} \exp(\phi_e(y_i, i, \mathbf{x}))$$

$$L(\mathbf{x}, \mathbf{y}) = \log \mathrm{P}(\mathbf{y}|\mathbf{x}) = \sum_{i=2}^{N} \phi_t(y_{i-1}, y_i) + \sum_{i=1}^{N} \phi_e(y_i, i, \mathbf{x}) - \log Z$$

In order to compute the loss function, we need two parts, $\sum_{i=2}^{N} \phi_t(y_{i-1}, y_i) + \sum_{i=1}^{N} \phi_e(y_i, i, \mathbf{x})$ which is called the gold score (unnormalized conditional log-probability), and $\log Z$.

(a) (**2 pts**) Now we are applying CRF to POS-tagging. The sentence, transition scores and emission scores are shown below. What is the gold score (unnormalized conditional log-probability) of this sentence? Show your work.

Sentence:	-START-	Atlanta	is	a	beautiful	city	-END-
POS tags:	START	n	v	det	adj	n	END

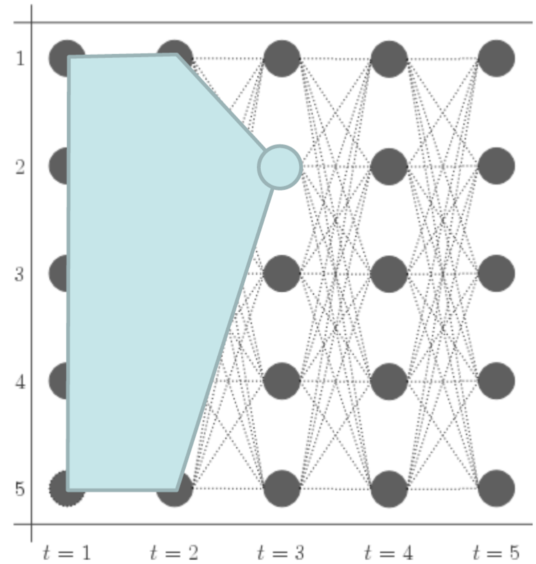
y_{i-1} \ y_i	START	n	v	det	adj	END
START	-0.1	0.63	0.67	-1.43	-1.96	-0.5
n	-0.9	1.24	0.76	0.41	0.23	0.65
v	-1.42	-0.24	-1.35	1.62	-1.76	1.28
det	-1.7	0.75	-0.65	-0.38	1.37	-1.93
adj	-1.76	1.66	0.04	-1.64	1.95	1.79
END	-1.55	-0.31	-1.46	-0.75	0.49	-1.35

Transition Scores: $\phi_t(y_{i-1}, y_i)$

x_i \ y_i	START	n	v	det	adj	END
-START-	1.97	-4.49	-3.29	3.16	-0.99	-0.81
Atlanta	0.96	-0.23	-1.15	-4.7	2.26	4.67
is	4.76	1.64	-1.44	-1.37	1.9	1.74
a	-3.86	-2.65	-1.37	0.11	3.1	0
beautiful	-4.72	3.23	-0.7	0.19	-2.78	-0.73
city	-1.12	2.72	-3.97	0.5	-3.22	1.96
-END-	-4.63	-2.23	-1.56	1.37	-4.48	-0.41

Emission Scores: $\phi_e(y_i, i, \mathbf{x}) = \phi_e(x_i, y_i)$

(b) (**3 pts**) For the $\log Z$ part, we know that $Z = \sum_{\mathbf{y}} \prod_{i=2}^{N} \exp(\phi_t(y_{i-1}, y_i)) \prod_{i=1}^{N} \exp(\phi_e(y_i, i, \mathbf{x}))$, but it is difficult to compute directly. Therefore, we need to use forward algorithm to compute iteratively and get the Z value eventually. The figure below shows the process of the forward algorithm.



Denote $\alpha_t(s_t)$ as sum of the unnormalized conditional probabilities ending at s_t up to time t. For example, in the figure above, the shaded part refers to $\alpha_3(2)$, which is the sum of the unnormalized conditional probabilities ending at $s_3 = 2$ up to time $t = 3$. Particularly, the initial should be $\alpha_1(s_1) = \exp(\phi_e(s, 1, \mathbf{x}))$ in our CRF.

Please write the recurrence formula for $\alpha_t(s_t)$ by using $\alpha_{t-1}(s_{t-1})$, ϕ_e and ϕ_t.

Given a sequence with length of N, write the relationship between Z and α.

In practice, however, we usually use log-probability instead. By defining $\gamma_t(s_t) = \log \alpha_t(s_t)$, please write the recurrence formula for $\gamma_t(s_t)$ by using $\gamma_{t-1}(s_{t-1})$, ϕ_e and ϕ_t, and show your work.