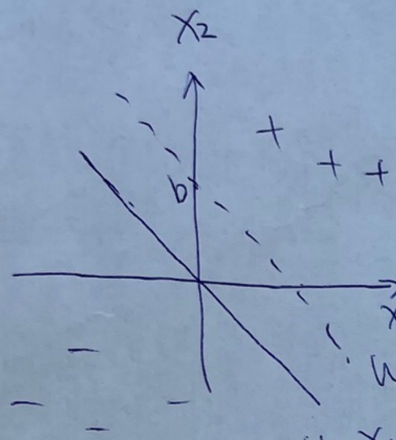
Linear Classification

bias term



$w_1x_1 + w_2x_2 + b = 0$

$w_1x_1 + w_2x_2 = 0$

$w = [w_1, w_2, \ldots w_3;]$

$w^T x$

$$f(x) = [\overset{x_1}{0.5}, \overset{x_2}{1.6}, \overset{x_3}{0.3}] \qquad w_1x_1 + w_2x_2 + w_3x_3 + b > 0$$

$$\downarrow$$

$$[0.5, 1.6, 0.3, \overset{x_0}{1}] \qquad w_1x_1 + w_2x_2 + w_3x_3 + \underset{b}{\underset{\parallel}{w_0}}\,\underset{1}{\underset{\parallel}{x_0}} > 0$$

$$w^T f(x) > 0$$

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argmax

$P(y=1|x) = 0.8$

$P(y=0|x) = 0.2$

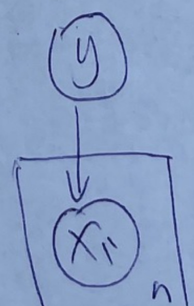
$\max P(y|x) = 0.8$

$\underset{y}{\operatorname{argmax}} P(y|x) = 1$

$\propto$

: proportion to :

plate notation



log

monotonically increasing

Maximum Likelihood

H H H T

$p \cdot p \cdot p \cdot (1-p) = p^3(1-p)$

$g(p)$ $\sum \log P(y_i)$.

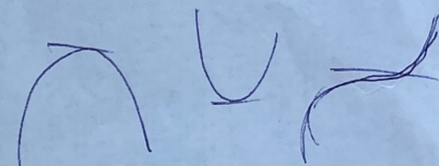
max $g(p)$
convex function

$$\frac{\partial g(p)}{\partial p} = 3 \cdot \frac{1}{p} + \frac{1}{1-p} \cdot (-1) = 0$$

$3(1-p) - p = 0$

$3 - 4p = 0$

$p = 3/4$



second derivative

$$\frac{\partial^2 g(p)}{\partial^2 p} = \frac{\partial}{\partial p}\left(\frac{\partial}{\partial p} g(p)\right) = 3 \cdot \underbrace{\frac{-1}{p^2}}_{<0} + \underbrace{\frac{-1}{(1-p)^2}}_{<0}$$

maxima / minima / saddle point

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logistic

$$\sigma(z) = \frac{1}{1+e^{-z}} = \frac{e^z}{e^z+1} = \frac{e^z}{1+e^z} = \frac{\exp(w^Tx)}{1+\exp(w^Tx)}$$

w^Tx

$$\log \frac{a}{b} = \log(a) - \log(b)$$

$$\log(\exp(c)) = c$$